Trigonometric Identities & Formulas

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Pythagorean Identities:

$$\cos^2\theta + \sin^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$\cot^2\theta + 1 = \csc^2\theta$$

Sum & Difference Identities:

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$$

Double-Angle Identities:

$$\sin 2\theta = 2\cos\theta\sin\theta$$

$$\begin{aligned}\cos 2\theta &= \cos^2\theta - \sin^2\theta \\ &= 1 - 2\sin^2\theta \\ &= 2\cos^2\theta - 1\end{aligned}$$

$$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$$

Laws of Sines & Cosines:

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$$

$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$

$$b^2 = a^2 + c^2 - 2ac\cos\beta$$

$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$

Half-Angle Identities:

$$\sin^2\alpha = \frac{1-\cos 2\alpha}{2} \quad \Rightarrow \quad \sin\tfrac{\beta}{2} = \pm\sqrt{\frac{1-\cos\beta}{2}}$$

$$\cos^2\alpha = \frac{1+\cos 2\alpha}{2} \quad \Rightarrow \quad \cos\tfrac{\beta}{2} = \pm\sqrt{\frac{1+\cos\beta}{2}}$$

$$\tan^2\alpha = \frac{1-\cos 2\alpha}{1+\cos 2\alpha} \quad \Rightarrow \quad \tan\tfrac{\beta}{2} = \pm\sqrt{\frac{1-\cos\beta}{1+\cos\beta}} = \frac{1-\cos\beta}{\sin\beta} = \frac{\sin\beta}{1+\cos\beta}$$

Product Formulas:

$$2\sin\alpha\sin\beta = \cos(\alpha-\beta) - \cos(\alpha+\beta)$$

$$2\cos\alpha\cos\beta = \cos(\alpha-\beta) + \cos(\alpha+\beta)$$

$$2\sin\alpha\cos\beta = \sin(\alpha+\beta) + \sin(\alpha-\beta)$$

$$2\cos\alpha\sin\beta = \sin(\alpha+\beta) - \sin(\alpha-\beta)$$

Factoring Formulas:

$$\sin\alpha + \sin\beta = 2\sin\tfrac{\alpha+\beta}{2}\cos\tfrac{\alpha-\beta}{2}$$

$$\sin\alpha - \sin\beta = 2\cos\tfrac{\alpha+\beta}{2}\sin\tfrac{\alpha-\beta}{2}$$

$$\cos\alpha + \cos\beta = 2\cos\tfrac{\alpha+\beta}{2}\cos\tfrac{\alpha-\beta}{2}$$

$$\cos\alpha - \cos\beta = -2\sin\tfrac{\alpha+\beta}{2}\sin\tfrac{\alpha-\beta}{2}$$

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