

Vector Function and Motion Formulas

Calculus III ~ Prof. Sally J. Keely, M.S.

Note: Unless otherwise indicated, all functions and vector functions are functions of t and primes denote derivatives with respect to t . Also $\vec{v} = \vec{r}'$, $\vec{a} = \vec{r}''$.

$$\text{Arc Length: } AL = \int_a^b \|\vec{v}\| dt$$

Curvature (κ) and Radius of Curvature (R):

$$\kappa = \left\| \frac{d\vec{T}}{ds} \right\| = \frac{\|\vec{T}'\|}{\|\vec{r}'\|} = \frac{\|\vec{v} \times \vec{a}\|}{\|\vec{v}\|^3} \quad R = \frac{1}{\kappa}$$

For special case of plane curve $y = f(x)$, $\vec{r} = x\hat{i} + f(x)\hat{j}$,

$$\kappa = \frac{|y''|}{(1 + (y')^2)^{3/2}} \quad \text{where the derivatives are with respect to } x.$$

Unit Tangent and Unit Normal Vectors: $\vec{T} = \frac{\vec{r}'}{\|\vec{r}'\|}$, $\vec{N} = \frac{\vec{T}'}{\|\vec{T}'\|}$

Unit Binormal Vector: $\vec{B} = \vec{T} \times \vec{N} = \frac{\vec{v} \times \vec{a}}{\|\vec{v} \times \vec{a}\|}$

Torsion: $\tau = -\frac{d\vec{B}}{ds} \cdot \vec{N} = -\frac{\vec{B}' \cdot \vec{N}}{\|\vec{v}\|} = \frac{(\vec{v} \times \vec{a}) \cdot \vec{a}'}{\|\vec{v} \times \vec{a}\|^2}$

Tangential and Normal Components of Acceleration:

$\vec{a} = a_T \vec{T} + a_N \vec{N}$ where,

$$a_T = \vec{a} \cdot \vec{T} = \frac{\vec{v} \cdot \vec{a}}{\|\vec{v}\|} \quad a_N = \vec{a} \cdot \vec{N} = \kappa \|\vec{v}\|^2 = \frac{\|\vec{v} \times \vec{a}\|}{\|\vec{v}\|}$$

and as a "check" $a_T^2 + a_N^2 = \|\vec{a}\|^2$